Noise Figure Minimization of RC Polyphase Filters

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Abstract **- Sideband suppression of RC polyphase filters is independent of the source and load impedances. This property is valid for any number of stages and any detuning between the stages. Consequently, noise figure minimization can be done independently of the adjustment of sideband suppression. A formula for noise figure of passive two-ports is obtained, and noise figure of polyphase filters is minimized. The noise figure is strongly dependent on the source resistance and capacitance. A formula for the lower bound of the noise figure of n stage polyphase filters is given. For a two-stage filter providing better than -25 dB sideband suppression over two octaves, a minimum noise figure of 10.18 dB was found.**

1 INTRODUCTION

RC polyphase filters (PPF) are widely used in communication systems [1]. Their typical applications are modulation and demodulation of single sideband signals. Due to their simplicity, they are popular in IF stages of application-specific integrated circuits. Noise in PPFs is qualitatively analyzed in [2], where the relation between resistor values of different stages is discussed for reducing noise. A pioneering paper on PPFs is [3]. Derivation of the structure and its sideband cancellation property are explained there in detail.

None of the mentioned publications consider terminations of PPFs for minimum noise figure. However, a PPF usually follows the input mixer of a receiver thus its contribution to the system noise is significant.

In this paper we show that the sideband suppression of an RC PPF is independent of the impedance of the source and the load under very general conditions (section 2). A formula for noise figure of passive two-ports is obtained (section 3) and applied for our case (section 4). As an example, noise figure of a two-stage RC PPF, designed for sideband suppression lower than -25 dB over two octaves, is minimized to 10.18 dB (section 5). Experimental verification is presented in section 6.

2 TRANSFER FUNCTIONS OF AN RC PPF

The one-stage RC PPF is an 8-node structure (Fig. 1) [3].

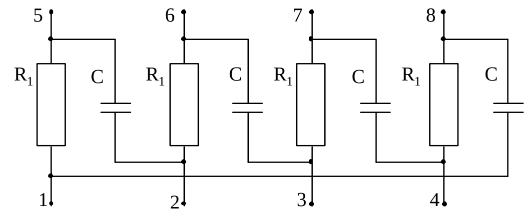

Figure 1: A one-stage RC PPF

In Fig. 1, all resistors and capacitors are identical. In a multistage filter, nodes 5-6-7-8 are connected to nodes 1-2-3-4 of the next stage, respectively. Resistor and capacitor values of different stages are not necessarily identical. In this paper, we consider the case when all capacitors are identical and only resistors of different stages may differ from each other.

Input and output ports are configured as shown in Fig. 2, nodes are in parentheses: 1 (1,3), 2 (2,4), 3 (5,7) and 4 (6,8). Thus our first goal is to obtain voltage gains from port 1 to ports 3 and 4, respectively.

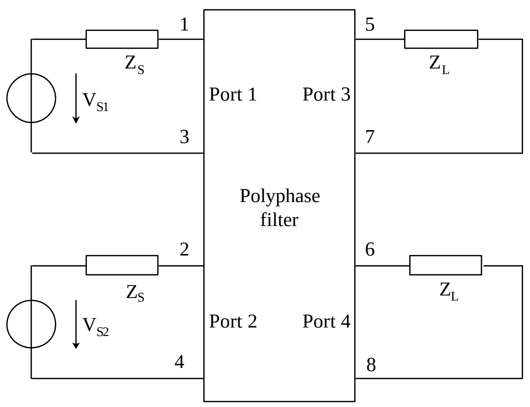

Figure 2: Excitation and loading of the PPF

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General formula for voltage gains of arbitrary number of stages is not known. For less than 30 dB sideband suppression over some frequency band, one or two stages are sufficient, thus we obtain voltage gains for one and two stage filters:

$$G_{V31,1} = \frac{Z_L}{j\omega CR_1 Z_L + 2R_1 + Z_L} \quad (1)$$

$$G_{V41,1} = -j\omega CR_1 \frac{Z_L}{j\omega CR_1 Z_L + 2R_1 + Z_L} \quad (2)$$

$$G_{V31,2} = Z_L \frac{1 + \omega^2 R_1 R_2 C^2}{D} \quad (3)$$

$$G_{V41,2} = -Z_L \frac{j\omega C(R_1 + R_2)}{D} \quad (4)$$

$$\begin{aligned} D &= 2R_1 + 2R_2 + Z_L \\ &+ j\omega C(4R_1R_2 + 3R_1Z_L + R_2Z_L) \\ &- \omega^2 C^2 R_1 R_2 Z_L \end{aligned} \quad (5)$$

where Eq. (1)-(2) show voltage gains for one stage, Eq. (3)-(5) for two stages, j is the imaginary unit, ω is the angular frequency and Z_L is the load impedance. $G_{V31,1}$ is the gain from port 1 to port 3 for 1 stage, etc. Resistor values in the first and second stages are denoted by R_1 and R_2, respectively.

The ratio of the voltage gains is

$$\frac{G_{V41,1}}{G_{V31,1}} = -j\omega CR_1 \quad (6)$$

$$\frac{G_{V41,2}}{G_{V31,2}} = \frac{-j\omega C(R_1 + R_2)}{1 + \omega^2 C^2 R_1 R_2} \quad (7)$$

where Eq. (6) is the ratio for one stage and Eq. (7) for two stages. The ratio of the voltage gains can also be expressed by the amplitude and phase match factors m and φ:

$$\frac{G_{V41}}{G_{V31}} = m \exp\left[j\left(\frac{\pi}{2} + \varphi \right) \right] \quad (8)$$

where m and φ are real quantities, $m \geq 0$. For perfect match, m=1 and $\varphi = \pm k\pi$ (k=0, 1,...). By Eq. (6)-(8), *amplitude and phase match factors are independent of the source and the load impedances*.

The sideband suppression can be expressed in terms of the amplitude and phase match factors:

$$S = 20\log \sqrt{\frac{[1 - m\cos(\varphi)]^2 + [m\sin(\varphi)]^2}{[1 + m\cos(\varphi)]^2 + [m\sin(\varphi)]^2}} \quad (9)$$

As a consequence, *the sideband suppression is also independent of the source and load impedances*. This property will appear important in noise minimization.

Sideband suppression usually depends on the difference between the two source impedances and that of the load impedances, thus we have to take care for the symmetry of terminations in realization.

3 NOISE OF LINEAR PASSIVE TWO-PORTS

Our goal is to minimize the noise figure of a PPF. Due to the required symmetry for terminating impedances, it is sufficient to consider noise figure for one input and one output, the others being properly terminated (Fig. 3). *Our model is equivalent to the impedance matrix model*.

The circuit in Fig. 3 is considered to be in thermal equilibrium, having common noise temperature T for all noise sources. Thermal noise in the two-port is modelled by the noise of Z_{out}. The two-port is excited by a source with noisy source impedance and loaded by a noiseless load impedance.

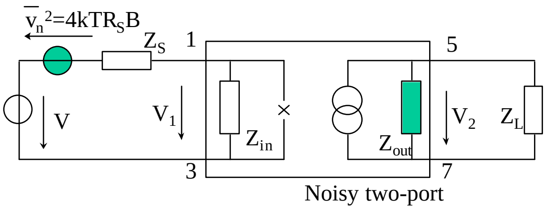

Figure 3: Z_S and Z_{out} are assumed to produce thermal noise. The two-port is modelled by Z_{in}, Z_{out} and the relation between V_2 and V_1. Ports 2 and 4 of Fig. 2 are terminated by Z_S and Z_L, respectively

The noise figure is defined as

$$F = \frac{SNR_{in}}{SNR_{out}} \quad (10)$$

where the signal to noise ratios are

$$SNR_{in} = \frac{P_{s\,in}}{P_{n\,in}} \qquad SNR_{out} = \frac{P_{s\,out}}{P_{n\,out}} \quad (11)$$

and we denoted the signal and the noise with subscripts s and n, respectively. The signal power at the input is, assuming sinusoidal signal with amplitude V_1:

$$P_{s\,in} = \frac{1}{2}\frac{|V_1|^2}{R_{in}} \quad (12)$$

The noise power at the input is the noise power coming from the impedance Z_S :

$$P_{n\,in} = \frac{4kTB}{R_{in}} \frac{R_s |Z_{in}|^2}{|Z_s + Z_{in}|^2} \quad (13)$$

Noise from port 2 in Fig. 2 is cancelled due to equipotentiality. In Eq. (13), k is the Boltzmann constant ($\approx 1.3807 \cdot 10^{-23}$ Joule/°K), T is the noise temperature and B is the noise bandwidth. The signal power at the output is

$$P_{s\,out} = \frac{1}{2}\frac{|V_2|^2}{R_L} \quad (14)$$

and the noise power is

$$P_{n\,out} = \frac{4kTB}{R_L} \frac{R_{out}|Z_L|^2}{|Z_{out} + Z_L|^2} \quad (15)$$

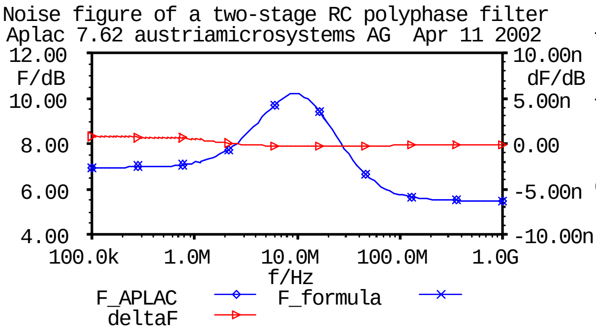

Fig. 4. Comparison of Eq. (16) to the analysis result

By combining the equations above, the noise figure is obtained:

$$\boxed{F = \frac{1}{|G_V|^2} \frac{|Z_s + Z_{in}|^2}{R_s |Z_{in}|^2} \frac{R_{out}|Z_L|^2}{|Z_{out} + Z_L|^2}} \quad (16)$$

In Eq. (16), $G_V = V_2 / V_1$ is the loaded voltage gain.

In Fig. 5 we compare the noise figure values obtained by the circuit analysis program APLAC [4], to those of Eq. (16). Agreement is good.

4 NOISE FIGURE MINIMIZATION

In this section, we use Eq. (16) for minimizing of the noise figure of a PPF. The noise minimum is found with respect to Z_S. Following this step, noise figure for the next stage (summing buffer) can be minimized with varying Z_L. In the last two steps, the invariance of the sideband suppression with respect to Z_S and Z_L is exploited.

Let us consider first the one-stage filter. Analytic expressions for $G_{V,1}$, Z_{in} and Z_{out} are

$$G_{V,1} = \frac{Z_L}{j\omega CR_1 Z_L + Z_L + 2R_1} \quad (17)$$

$$Z_{in,1} = \frac{j\omega CR_1 Z_L + Z_L + 2R_1}{j\omega CZ_L + j\omega CR_1 + 1} \quad (18)$$

$$Z_{out,1} = \frac{j\omega CR_1 Z_S + Z_S + 2R_1}{j\omega CZ_S + j\omega CR_1 + 1} \quad (19)$$

With these expressions substituted into Eq. (16), the noise figure at $\omega_1 = 1/CR_1$ is:

$$F_1 = \frac{R_S^2 + 2R_S R_1 + X_S^2 - 2X_S R_1 + 2R_1^2}{R_1 R_S} \quad (20)$$

where $Z_S = R_S + jX_S$. Noise figure at ω_1 is independent of Z_L as expected. The noise figure has a minimum at

$$R_S = X_S = R_1 \quad (21)$$

However, in IC realization, inductive termination is not allowed. With $X_S = 0$, the noise figure has a minimum at

$$R_S = \sqrt{2} R_1 \quad (22)$$

Its value is

$$\min(F_1) = 2 + 2\sqrt{2} \quad (23)$$

that is, 6.838 dB.

Now let us continue with a two-stage filter. Substituting the expressions for $G_{V,2}$, $Z_{in,2}$ and $Z_{out,2}$ into Eq. (16), the noise figure at center frequency $\omega_C = 1/C\sqrt{R_1 R_2}$ is obtained as follows, observing that ω_C corresponds to the local maximum of the sideband suppression:

$$F_2 = \frac{1}{4}\frac{2R_s^2(R_1+3R_2)+R_s(R_1^2+14R_1R_2+R_2^2)+4R_1R_2(3R_1+R_2)}{R_sR_1R_2} \quad (24)$$

where all terminations are considered as resistive. The noise figure is independent of the load as before. The noise figure has a minimum at

$$R_S = \sqrt{\frac{2R_1R_2(3R_1+R_2)}{R_1+3R_2}} \quad (25)$$

We note that in case of $R_1=R_2$, Eq. (25) reduces to Eq. (22), and

$$\min(F_2) = 4 + 4\sqrt{2} \quad (26)$$

that is, 9.85 dB. This is the lower bound of the noise figure of two-stage RC PPFs.

By comparison of Eq. (23) and (26), it is easy to find the general formula for lower bound of the noise figure of n stages:

$$\boxed{\min(F_n) = 2^n(1+\sqrt{2})} \quad (27)$$

This noise figure occurs at the frequency of perfect amplitude match when all resistors of different stages are identical. We checked the validity of Eq. (27) by computer simulation for n=1,2...5 successfully.

5 EXAMPLE: A TWO-STAGE DETUNED POLYPHASE FILTER

As an example, the noise figure of a two-stage detuned PPF is minimized. Input data for the design is the capacitance value C=8 pF (due to the estimated silicon area), the center frequency f_c=10 MHz and the sideband suppression at center frequency S_{max}=-25 dB. The following equations are solved for R_1 and R_2:

$$f_C = \frac{1}{2\pi C\sqrt{R_1R_2}} \quad (28)$$

$$S_{max} = 20\log\left(\frac{R_1+R_2-2\sqrt{R_1R_2}}{R_1+R_2+2\sqrt{R_1R_2}}\right) \quad (29)$$

where Eq. (29) follows from Eq. (7)-(9) and the condition for local maximum of S.

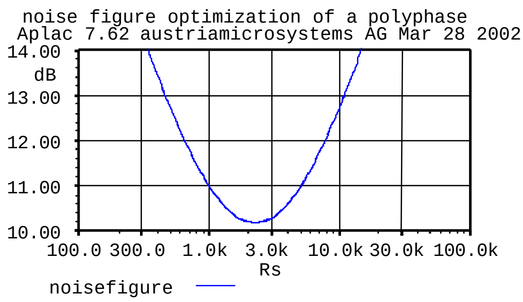

Fig. 5. Noise figure as a function of the source resistance, when source capacitance is zero

Eq. (28)-(29) have two pairs of solutions, one of them is R_1=1.227 kΩ, R_2=3.226 kΩ. Applying Eq. (25) and (24) for noise matching, the source resistance for minimum noise figure is R_S=2.239 kΩ, and the noise figure is F_2=10.18dB. Noise figure at the center frequency as a function of the source resistance and capacitance are given in Fig. 5 and 6, respectively.

It can be shown that f_c and S_{max} determine filter bandwidth. Therefore the capacitance value as an input variable cannot be replaced by an equation for bandwidth.

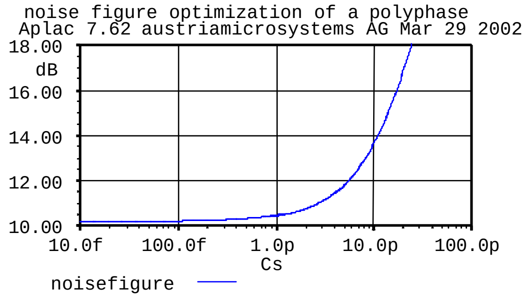

Fig. 6. Noise figure as a function of source capacitance, when source resistance is tuned to minimum noise figure at zero source capacitance

6 MEASUREMENTS

This is a verification of the theoretical sideband suppression curve of a PPF and also a verification of the statement that the sideband suppression is independent of the source resistance (Fig. 7).

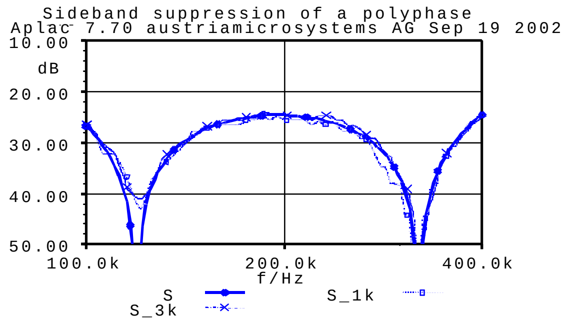

Fig. 7. Theoretical (S) and measured (S_1k, S_3k) sideband suppression of a two-stage PPF

Fig. 7 has been obtained using a realized two-stage detuned PPF with circuit element values R_1= 490 Ω, R_2=1291 Ω and C=1000 pF. Node 1 was excited with an opamp output in series with a resistor R_S. Node 3 was grounded. Nodes 2 and 4 were terminated also by R_S. Output ports were symmetrically terminated by differential amplifiers. All opamps are LM318. The voltage gains have been measured by a HP3575A Gain-Phase Meter. From the voltage gains sideband suppression has been calculated using Eq. (9).

As shown in Fig. 7, R_S=1 kΩ and R_S=3 kΩ was applied. Coincidence between theoretical and measured curves is good, also demonstrating that sideband suppression is really independent of the source resistance.

Thus the basis of the main statement of this paper is verified.

7 CONCLUSIONS

Some results on noise figure minimization of RC PPFs have been obtained. In section 2 voltage transfer functions are given. It is shown that the sideband suppression is independent of the source (and load) terminations, thus noise figure can be minimized without degradation of the sideband suppression. In section 3 a formula for noise figure of passive two-ports is shown. In section 4 noise figure of PPFs are minimized. In section 5 we apply our results in designing and noise matching of a two-stage detuned PPF. In section 6 the independence of the sideband suppression of the source resistance is verified experimentally.

The independence of the sideband suppression from source and load is proved here for one- and two stages. This property was observed as generally valid for arbitrary number of stages, however.

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